MATH 1A - HOW TO SIMPLIFY INVERSE TRIG FORMULAS

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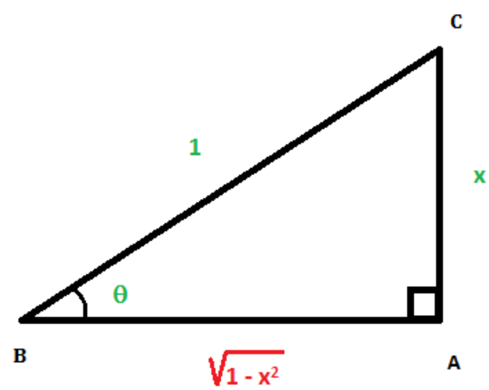
Sample Problem (1.6.65) : Show $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$

1. How to write out your answer

Let $\theta = \sin^{-1}(x)$ (then $\sin(\theta) = x$).

1A/Handouts/Triangle.png



Then:

$$\cos(\sin^{-1}(x)) = \cos(\theta) = \frac{AB}{BC} \overset{PYTH}{=} \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

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2. DETAILED VERSION

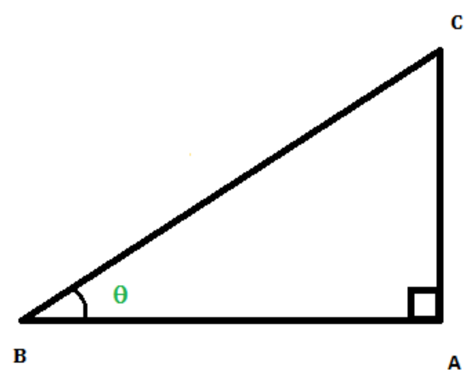
First of all, let $\theta = \sin^{-1}(x)$. Then $\sin(\theta) = x$ (remember that when you're putting $\sin^{-1}$ on the other side of the equality, you remove the $^{-1}$).

Our goal is to evaluate $\cos(\sin^{-1}(x)) = \cos(\theta)$ (because $\sin^{-1}(x) = \theta$). Once we compute $\cos(\theta)$, we're done!.

Now, since we know that $\sin(\theta) = x$, the trick is to draw the easiest right triangle you can think of with the property that $\sin(\theta) = x$.

First, let's draw a right triangle ABC. We'll complete it in several steps.

1A/Handouts/Triangle1.png



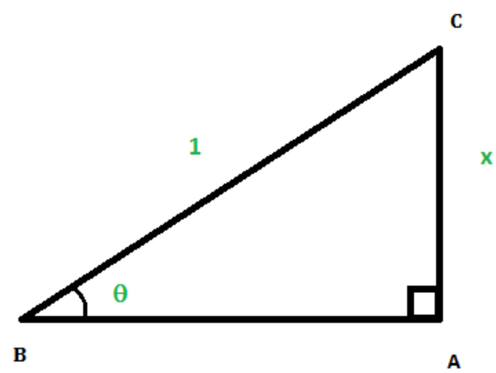
Looking at the triangle, we know that $\sin(\theta) = \frac{\text{OPP}}{\text{HYP}} = \frac{AC}{BC}$. On the other hand, we want $\sin(\theta) = x$, so $\frac{AC}{BC} = x$.

For example, choose $AC = x$ and $BC = 1$.

IMPORTANT NOTE: It will **ALWAYS** be the case that one side is x and the other one 1.

So our triangle looks like as follows:

1A/Handouts/Triangle2.png



We're almost done! Remember that our goal is to compute $\cos(\theta)$, and using the above triangle, we can do precisely that!

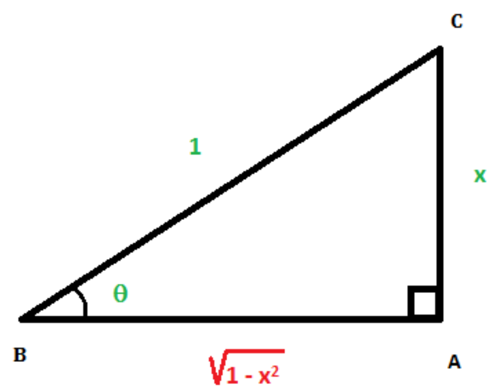
$$\cos(\theta) = \frac{AB}{BC} = \frac{AB}{1} = AB$$

What is AB? Using the Pythagorean theorem, we know that:

$$\begin{aligned} AC^2 + AB^2 &= BC^2 \\ x^2 + AB^2 &= 1 \\ AB^2 &= 1 - x^2 \\ AB &= \sqrt{1 - x^2} \end{aligned}$$

So we can complete our picture as follows:

1A/Handouts/Triangle.png



And finally, putting everything together, we get:

$$\cos(\sin^{-1}(x)) = \cos(\theta) = AB = \sqrt{1-x^2}$$

And we're done!

3. Another solution

Starting with the identity $(\cos(\theta))^2 + (\sin(\theta))^2 = 1$, we let $\theta = \sin^{-1}(x)$, and we get:

$$\begin{aligned}
\left(\sin(\sin^{-1}(x))\right)^2 + \left(\cos(\sin^{-1}(x))\right)^2 &= 1 \\
x^2 + \left(\cos(\sin^{-1}(x))\right)^2 &= 1 \\
\left(\cos(\sin^{-1}(x))\right)^2 &= 1 - x^2 \\
\cos(\sin^{-1}(x)) &= \pm\sqrt{1-x^2}
\end{aligned}$$

Now the question is: Which do we choose, $\sqrt{1-x^2}$, or $-\sqrt{1-x^2}$, and this requires some thinking!
The thing is: We defined $\sin^{-1}(x)$ to have range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ so, $\cos(\sin^{-1}(x))$ has range $[0,1]$, and is in particular ≥ 0 (see picture below for more clarification).

So, since $\cos(\sin^{-1}(x)) \geq 0$, the answer **has** to be $\sqrt{1-x^2}$.

Note: Feel free to use this solution on your exam, but you have to justify why your final answer is ≥ 0.

1A/Handouts/Theta.png